C183 HW4

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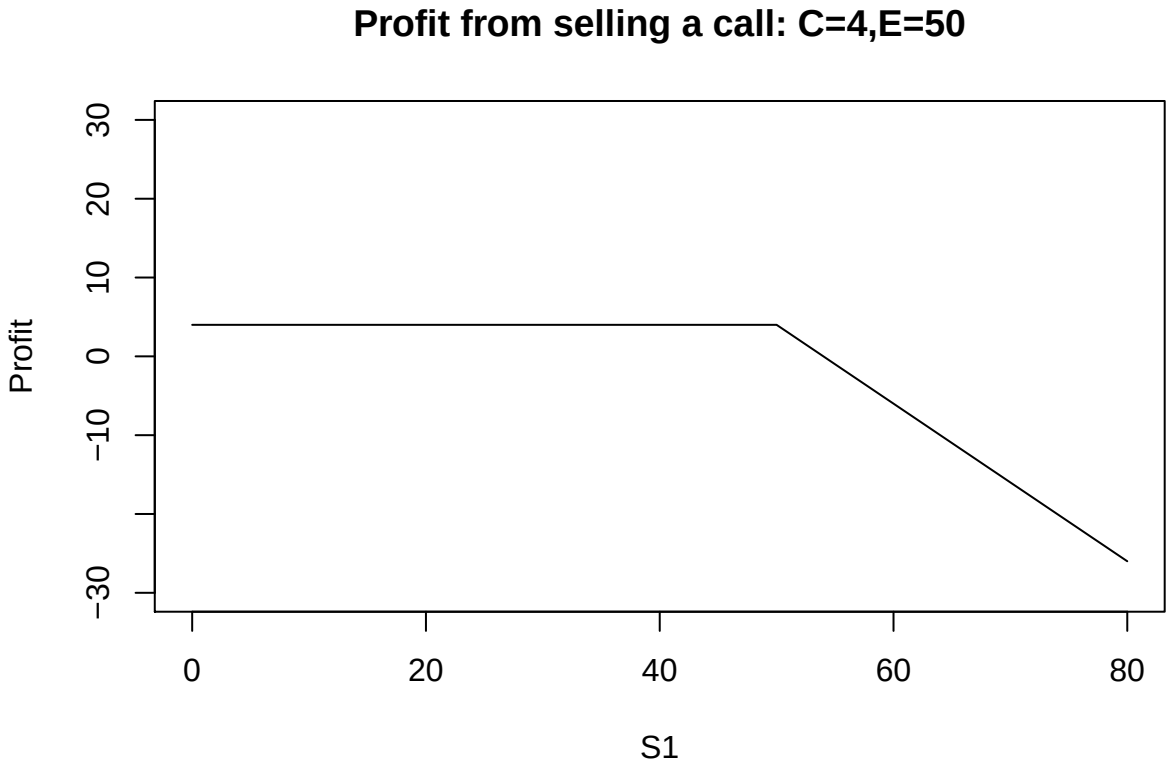
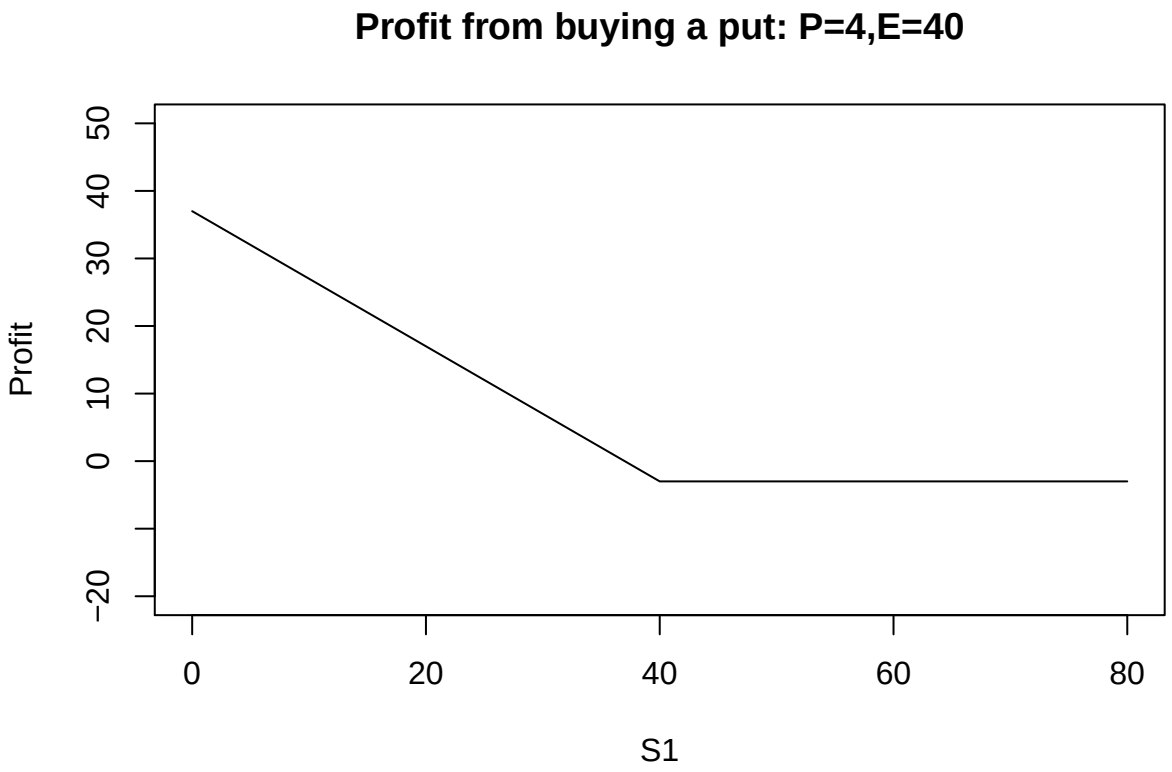
Exercise 1

```
S1 <- seq(0,80,by=1)
C <- 4
E <- 50
x2 <- ifelse(S1 > E, E-S1+C, C)
plot(S1,x2,ylim=c(-30,30),type="l",
     main="Profit from selling a call: C=4,E=50",ylab="Profit",xlab="S1")
```



Exercise 2

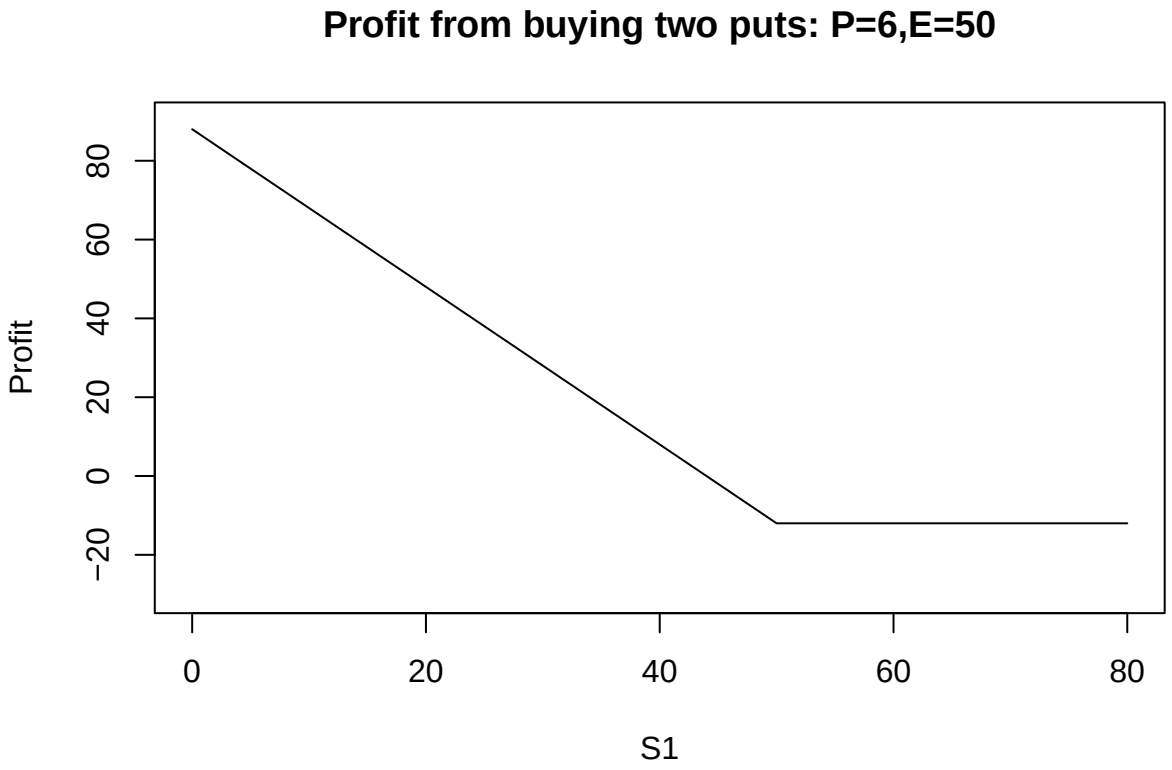
```
S1 <- seq(0,80,by=1)
P <- 3
E <- 40
x2 <- ifelse(S1 < E, E-S1-P,-P)
plot(S1,x2,ylim=c(-20,50),type="l",
     main="Profit from buying a put: P=4,E=40",ylab="Profit",xlab="S1")
```



Exercise 3

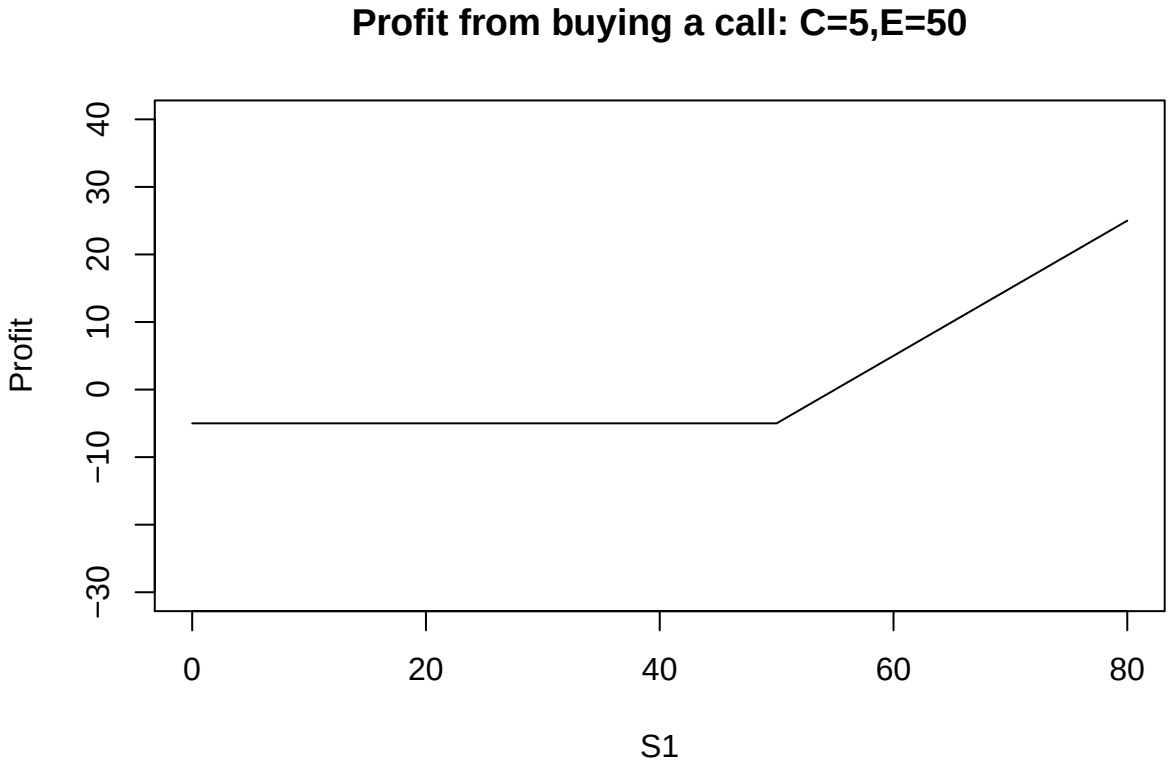
a

```
S1 <- seq(0,80,by=1)
P <- 6
E <- 50
x2 <- ifelse(S1 < E, (E-S1-P)*2,-P*2)
plot(S1,x2,ylim=c(-30,90),type="l",
     main="Profit from buying two puts: P=6,E=50",ylab="Profit",xlab="S1")
```



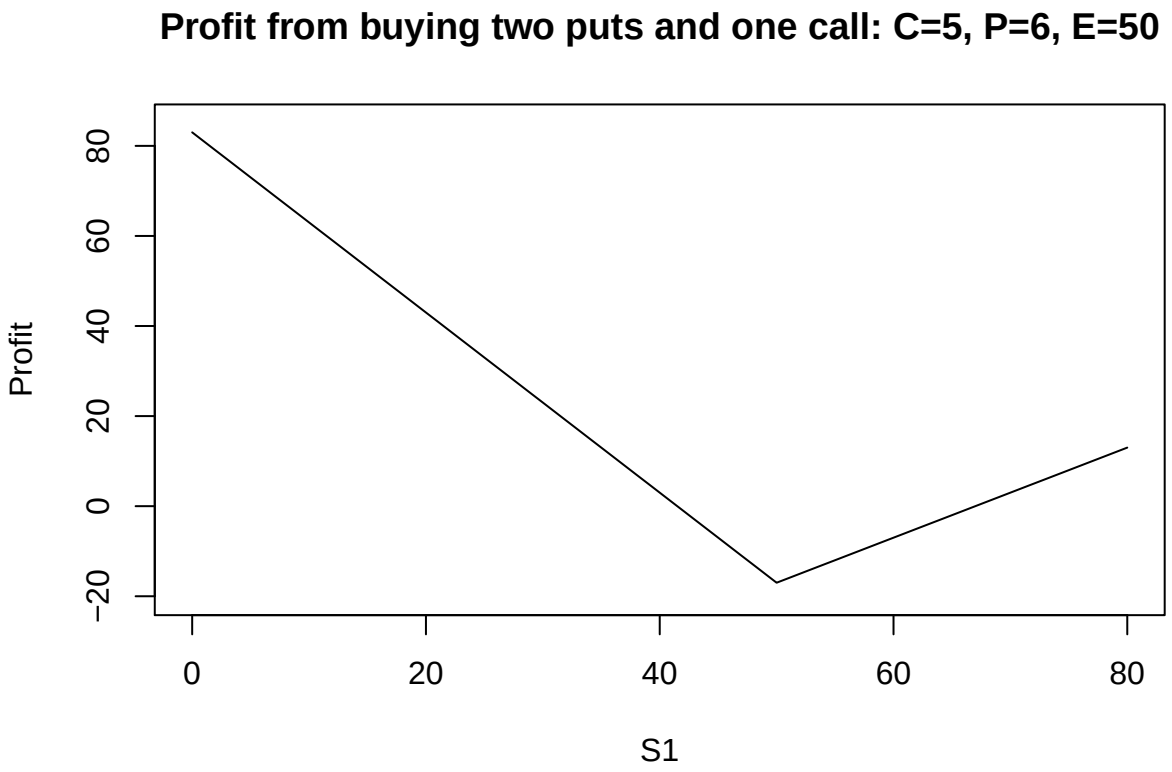
b

```
S1 <- seq(0,80,by=1)
C <- 5
E <- 50
x2 <- ifelse(S1 > E, S1-E-C,-C)
plot(S1,x2,ylim=c(-30,40),type="l",
     main="Profit from buying a call: C=5,E=50",ylab="Profit",xlab="S1")
```



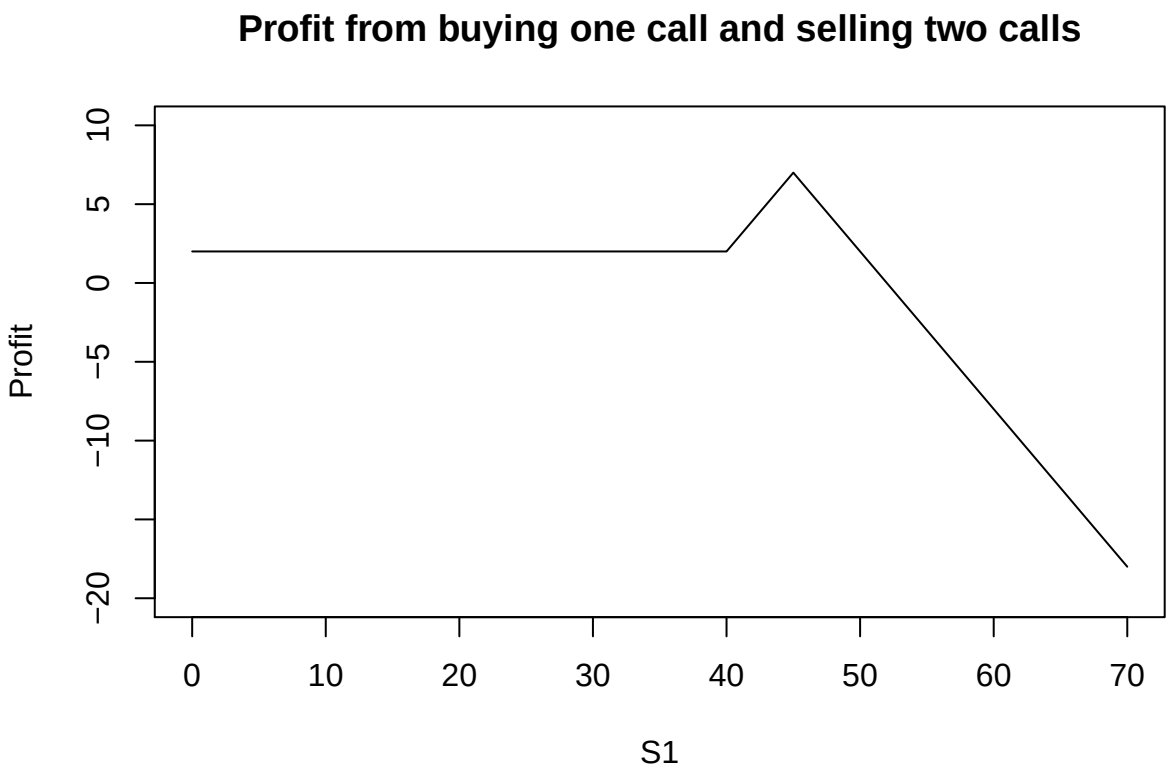
c

```
S1 <- seq(0,80,by=1)
P <- 6
E <- 50
C <- 5
x1 <- ifelse(S1 > E, S1-E-C,-C)
x2 <- ifelse(S1 < E, (E-S1-P)*2,-P*2)
x3 <- x1+x2
plot(S1,x3,ylim=c(-20,85),type="l",
     main="Profit from buying two puts and one call: C=5, P=6, E=50",ylab="Profit",xlab="S1")
```



Exercise 4

```
S1 <- seq(0,70,by=1)
sC <- 5
sE <- 45
bC <- 8
bE <- 40
x1 <- ifelse(S1 > bE, S1-bE-bC,-bC)
x2 <- ifelse(S1 > sE, -(S1-sE-sC)*2,sC*2)
x3 <- x1+x2
plot(S1,x3,ylim=c(-20,10),type="l",
     main="Profit from buying one call and selling two calls",ylab="Profit",xlab="S1")
```



Exercise 5

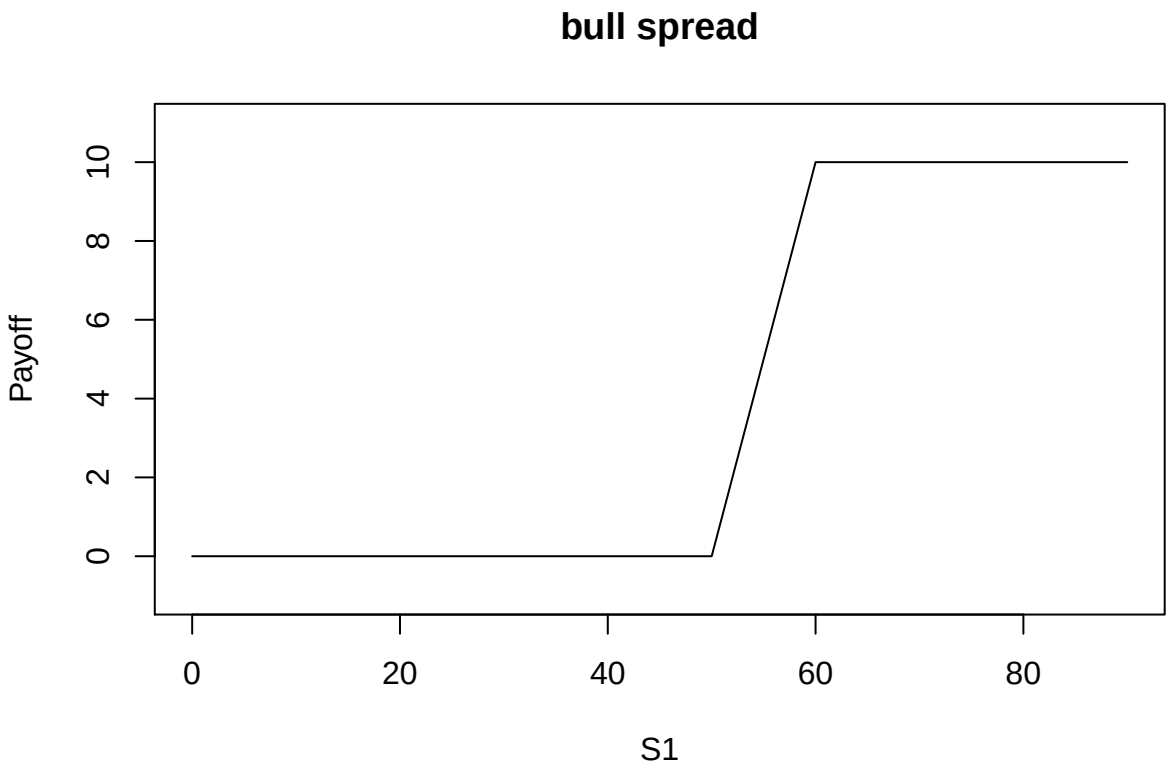
a

S_T	buy call	sell call	buy put	sell put	Total
$S_T > E_2$	$S_T - E_1$	$E_2 - S_T$	0	0	$E_2 - E_1$
$E_1 < S_T < E_2$	$S_T - E_1$	0	$E_2 - S_T$	0	$E_2 - E_1$
$S_T < E_1$	0	0	$E_2 - S_T$	$S_T - E_1$	$E_2 - E_1$

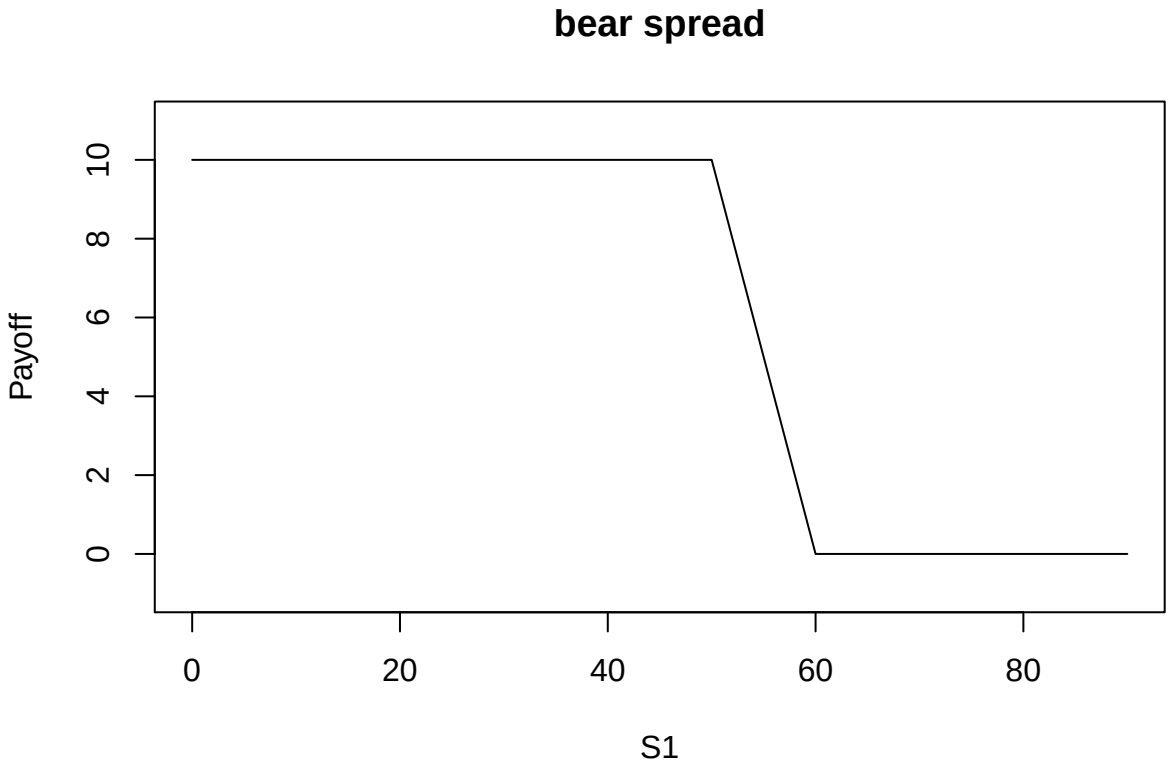
b

```
S1 <- seq(0,90,by=1)
sCE <- 60
bCE <- 50
sPE <- 50
bPE <- 60
x1 <- ifelse(S1 > bCE, S1-bCE,0)
x2 <- ifelse(S1 > sCE, -(S1-sCE),0)
x3 <- ifelse(S1 < sPE, -(sPE-S1),0)
x4 <- ifelse(S1 < bPE, bPE-S1,0)
x5 <- x1+x2+x3+x4
bull <- x1+x2
bear <- x3+x4
```

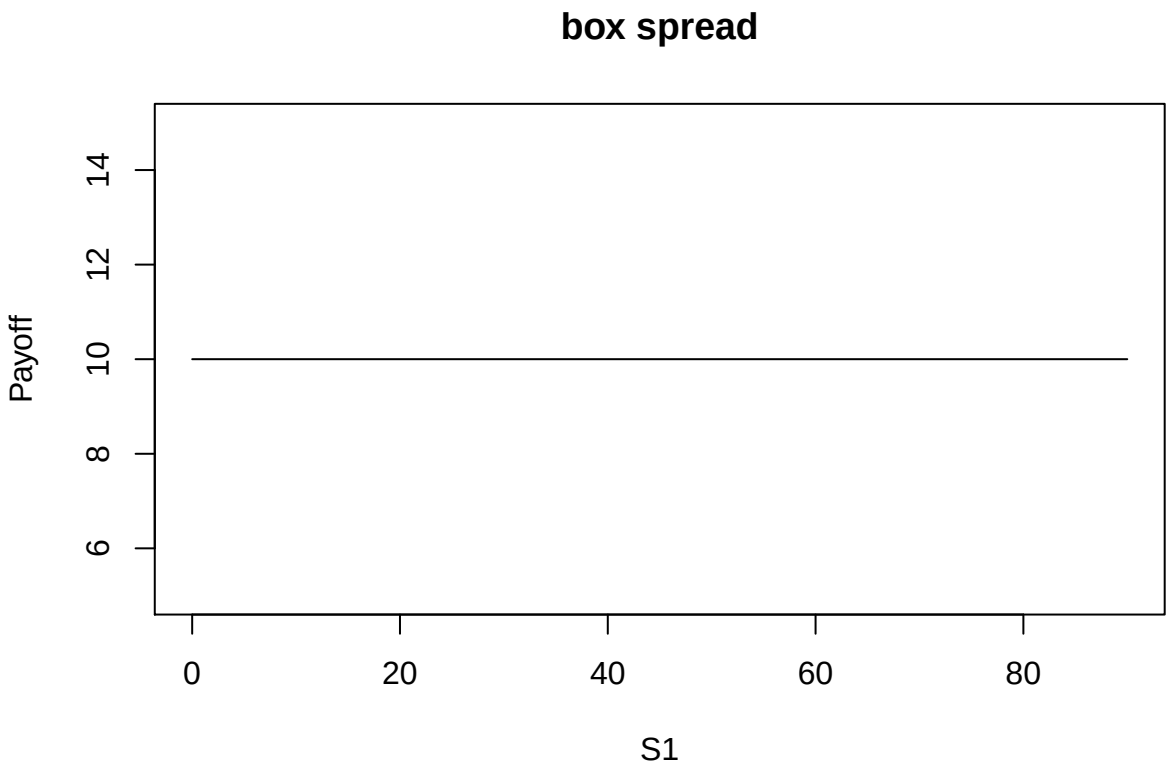
```
plot(S1,bull,ylim=c(-1,11),type="l",
     main="bull spread",ylab="Payoff",xlab="S1")
```




```
plot(S1,bear,ylim=c(-1,11),type="l",
     main="bear spread",ylab="Payoff",xlab="S1")
```



```
plot(S1,x5,ylim=c(5,15),type="l",
     main="box spread",ylab="Payoff",xlab="S1")
```



Exercise 6

a

k should be round up to 6

```
k <- log(60/((1/1.2)^10 * 50 ))/log(1.2/(1/1.2));k
```

```
## [1] 5.5
```

b

```
u <- 1.2
d <- 1/1.2
S <- 50
E <- 60
Su10 <- 50 * u^10
Su9d1 <- 50 * u^9* d
Su8d2 <- 50 * u^8* d^2
Su7d3 <- 50 * u^7* d^3
Su6d4 <- 50 * u^6* d^4
Su5d5 <- 50 * u^5* d^5
Su4d6 <- 50 * u^4* d^6
Su3d7 <- 50 * u^3* d^7
```

```
Su2d8 <- 50 * u^2* d^8
Su1d9 <- 50 * u^1* d^9
Sd10 <- 50 * d^10
period10 <- c(Su10,Su9d1,Su8d2,Su7d3,Su6d4,Su5d5,Su4d6,Su3d7,Su2d8,Su1d9,Sd10)
period10
```

```
##  [1] 309.586821 214.990848 149.299200 103.680000  72.000000  50.000000
##  [7]  34.722222  24.112654  16.744899  11.628402   8.075279
```

c

```
c10 <- ifelse(period10 > E, period10-E , 0)
c10
```

```
##  [1] 249.5868 154.9908  89.2992  43.6800  12.0000   0.0000   0.0000
##  [8]   0.0000   0.0000   0.0000   0.0000
```

d

1

```
p <- (1+0.1-d)/(u-d)
pprime <- (p*u)/(1+0.1)
50*pbinom(5,10,pprime, lower.tail=FALSE) -
  (60/(1+0.1)^10)*pbinom(5,10, p, lower.tail=FALSE)
```

```
## [1] 27.48628
```

2

```
p <- (1 + 0.1 - d) / (u-d)

pro <- c(p^10 * choose(10,0), p^9 * (1-p) * choose(10,1), p^8 * (1-p)^2 * choose(10,2),
         p^7 * (1-p)^3 * choose(10,3), p^6 * (1-p)^4 * choose(10,4), p^5 * (1-p)^5* choose(10,5),
         p^4 * (1-p)^6* choose(10,6),p^3 * (1-p)^7* choose(10,7),p^2 * (1-p)^8* choose(10,8),
         p^1 * (1-p)^9* choose(10,9), (1-p)^10* choose(10,10))

sum(pro * c10) / (1+0.1)^10
```

```
## [1] 27.48628
```

Exercise 7

```
u <- 1+0.06
d <- 1-0.05
S <- 50
E <- 51
t<-1/4
rp <- 0.05
```

```
p <- (exp(rp*t)-d)/(u-d)

Su <- S * u
Sd <- S * d
Sud <- S * u * d
Suu <- S * u * u
Sdd <- S * d * d

bioptions <- matrix(rep("",15),nrow=5,ncol=3)

Cuu <- max(Suu-E,0)
Cud <- max(Sud-E,0)
Cdd <- max(Sdd-E,0)
Cu <- (Cuu*p + Cud*(1-p))/(exp(rp*t))
Cd <- (Cud*p + Cdd*(1-p))/(exp(rp*t))
C <- (Cu*p + Cd*(1-p))/(exp(rp*t))

bioptions[3,1] <- paste("S0=",S,", C=",round(C,4))
bioptions[2,2] <- paste("Su=",Su,", Cu=",round(Cu,3));
bioptions[4,2] <- paste("Sd=",Sd,", Cd=",round(Cd,3))
bioptions[1,3] <- paste("Suu=",Suu,", Cuu=",Cuu)
bioptions[3,3] <- paste("Sdu=",Sud,", Cud=",Cud)
bioptions[5,3] <- paste("Sdd=",Sdd,", Cdd=",Cdd)
bioptions
```

```
##      [,1]                 [,2]                [,3]
## [1,] ""                   ""                  "Suu= 56.18 , Cuu= 5.18"
## [2,] ""                   "Su= 53 , Cu= 2.91" ""
## [3,] "S0= 50 , C= 1.6351" ""                  "Sdu= 50.35 , Cud= 0"
## [4,] ""                   "Sd= 47.5 , Cd= 0"  ""
## [5,] ""                   ""                  "Sdd= 45.125 , Cdd= 0"
```

Exercise 8

```
Puu <- max(E-Suu,0)
Pud <- max(E-Sud,0)
Pdd <- max(E-Sdd,0)
Pu <- (Puu*p + Pud*(1-p))/(exp(rp*t))
Pd <- (Pud*p + Pdd*(1-p))/(exp(rp*t))
P <- (Pu*p + Pd*(1-p))/(exp(rp*t))

bioptions[3,1] <- paste("S0=",S,", P=",round(P,3))
bioptions[2,2] <- paste("Su=",Su,", Pu=",round(Pu,3));
bioptions[4,2] <- paste("Sd=",Sd,", Pd=",round(Pd,3))
bioptions[1,3] <- paste("Suu=",Suu,", Puu=",Puu)
bioptions[3,3] <- paste("Sdu=",Sud,", Pud=",round(Pud,3))
bioptions[5,3] <- paste("Sdd=",Sdd,", Pdd=",Pdd)
bioptions
```

```
##      [,1]                 [,2]                  [,3]
## [1,] ""                   ""                    "Suu= 56.18 , Puu= 0"
## [2,] ""                   "Su= 53 , Pu= 0.277"  ""
```

```
## [3,] "S0= 50 , P= 1.376" ""                      "Sdu= 50.35 , Pud= 0.65"
## [4,] ""                  "Sd= 47.5 , Pd= 2.866" ""
## [5,] ""                  ""                     "Sdd= 45.125 , Pdd= 5.875"
```

```
C + E/(exp(rp*(1/2)));P+S # t =1/2
```

```
## [1] 51.37588
```

```
## [1] 51.37588
```

```
Cu + E/(exp(rp*(1/4)));Pu+Su # t =1/4
```

```
## [1] 53.27674
```

```
## [1] 53.27674
```

```
Cd + E/(exp(rp*(1/4)));Pd+Sd
```

```
## [1] 50.36647
```

```
## [1] 50.36647
```

```
Cuu + E/(exp(rp*0));Puu+Suu # t = 0
```

```
## [1] 56.18
```

```
## [1] 56.18
```

```
Cud + E/(exp(rp*0));Pud+Sud
```

```
## [1] 51
```

```
## [1] 51
```

```
Cdd + E/(exp(rp*0));Pdd+Sdd
```

```
## [1] 51
```

```
## [1] 51
```

Exercise 9

If the put option were American, we need to calculate the value of the put at each node and compare it to the payoff from early exercise. The put value at any node will be the greater beteen these two values.

```
early0 <- max(E-S,0)
earlyu <- max(E-Su,0)
earlyd <- max(E-Sd,0)
earlyuu <- max(E-Suu,0)
earlyud <- max(E-Sud,0)
earlydd <- max(E-Sdd,0)
bioptions[3,1] <- paste("P=",round(P,3),", EE=",early0)
bioptions[2,2] <- paste("Pu=",round(Pu,3),", EE=",earlyu);
bioptions[4,2] <- paste("Pd=",round(Pd,3),", EE=",earlyd)
bioptions[1,3] <- paste("Puu=",Puu,", EE=",earlyuu)
bioptions[3,3] <- paste("Pud=",round(Pud,3),", EE=",round(earlyud,3))
bioptions[5,3] <- paste("Pdd=",Pdd,", EE=",earlydd)
bioptions
```

```
##      [,1]              [,2]                  [,3]
```

```
## [1,] ""                  ""                      "Puu= 0 , EE= 0"
## [2,] ""                  "Pu= 0.277 , EE= 0"     ""
## [3,] "P= 1.376 , EE= 1" ""                       "Pud= 0.65 , EE= 0.65"
## [4,] ""                  "Pd= 2.866 , EE= 3.5"   ""
## [5,] ""                  ""                      "Pdd= 5.875 , EE= 5.875"
```

```
P <- (Pu*p + earlyd*(1-p))/(exp(rp*t));P
```

```
## [1] 1.645603
```

We can see that when t=1 and stock price goes down, the price for put option is 2.866. However, if we can early exercise the put option, the payoff is 3.5, which is greater than 2.866. Hence it is optimal to early exercise the put option at that node. In addition, if we had early exercised the put option at t=1, then the put price at t=0 should also be changed based on the put price at t=1. Eventually, the put price at t=0 should be $1.645603